Solutions by
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GCSE (9-1) Grade 8/9 Completing the Square

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Show all your working out**

Information

- The total mark for this paper is 92.
- The marks for **each** question are shown in brackets.
  – use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed

Advice

- Read each question carefully before you start to answer it
- Attempt every question
- Check your answers if you have time at the end

$8 \div 2 = 4$ $\quad 4^2 = 16$

1. (a) Write $x^2 - 8x + 20$ in the form $(x + a)^2 + b$ where a and b are integers

$(x-4)^2 + 20 - 16$

$(x-4)^2 + 4$

.....

(2 marks)

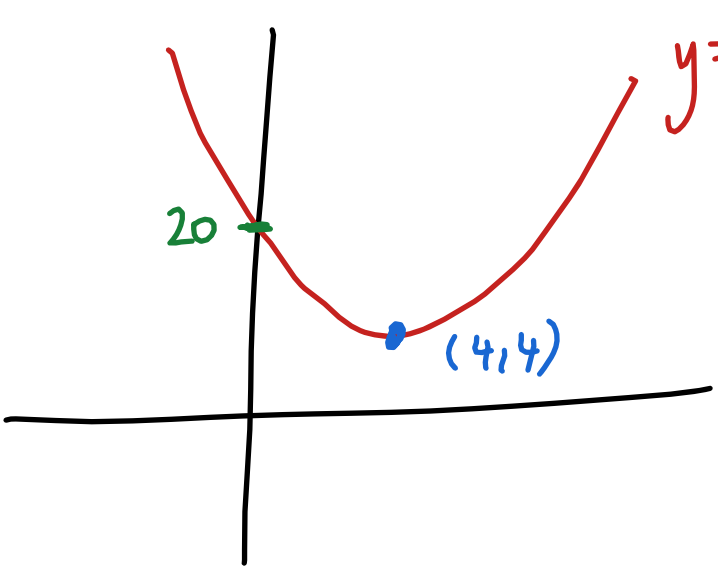
(b) Hence or otherwise write down the coordinates of the turning point of the graph $y = x^2 - 8x + 20$

$(+4, 4)$

.....

(1 mark)

(c) Hence sketch the curve $y = x^2 - 8x + 20$



.....

(2 marks)

2. (a) Write $x^2 - 10x - 1$ in the form $(x + a)^2 + b$ where a and b are integers

$(x-5)^2 - 1 - 25$

$(x-5)^2 - 26$

.....

(2 marks)

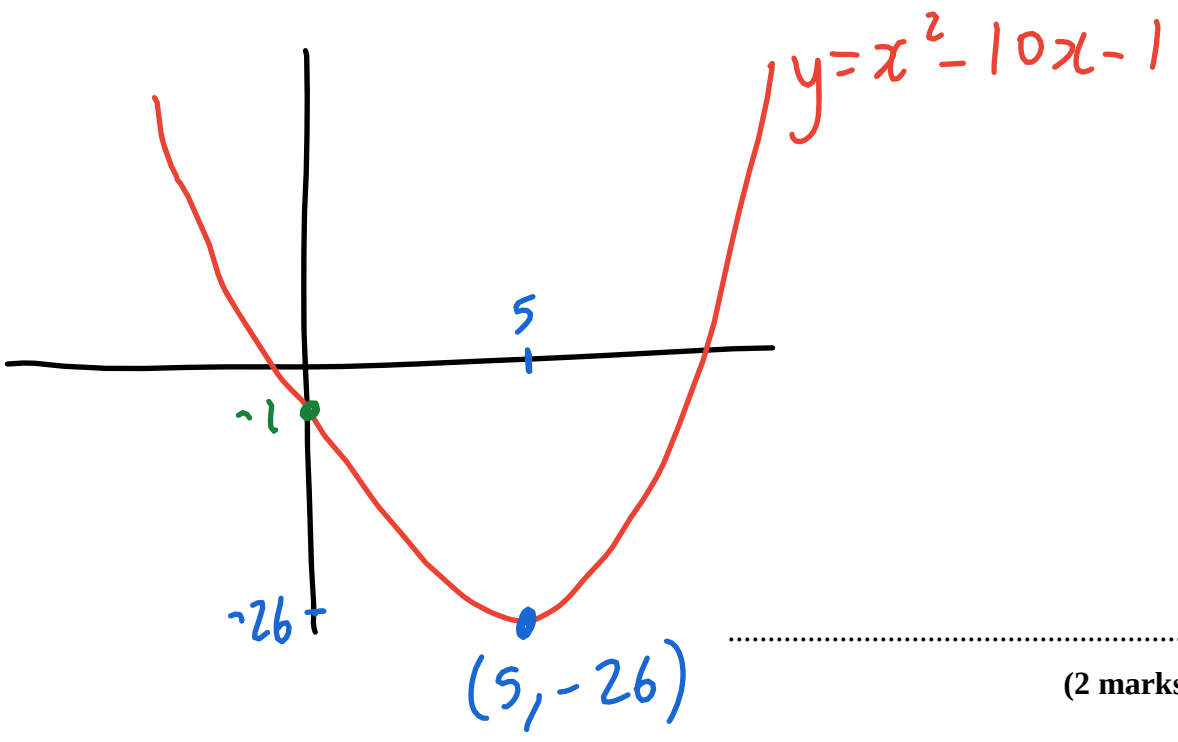
(b) Hence or otherwise write down the coordinates of the turning point of the graph $y = x^2 - 10x - 1$

$(5, -26)$

.....

(1 mark)

(c) Hence sketch the curve $y = x^2 - 10x - 1$



.....

(2 marks)

3. (a) Write $x^2 - 4x - 6$ in the form $(x + a)^2 + b$ where a and b are integers

$(x-2)^2 - 6 - 4$

$(x-2)^2 - 10$

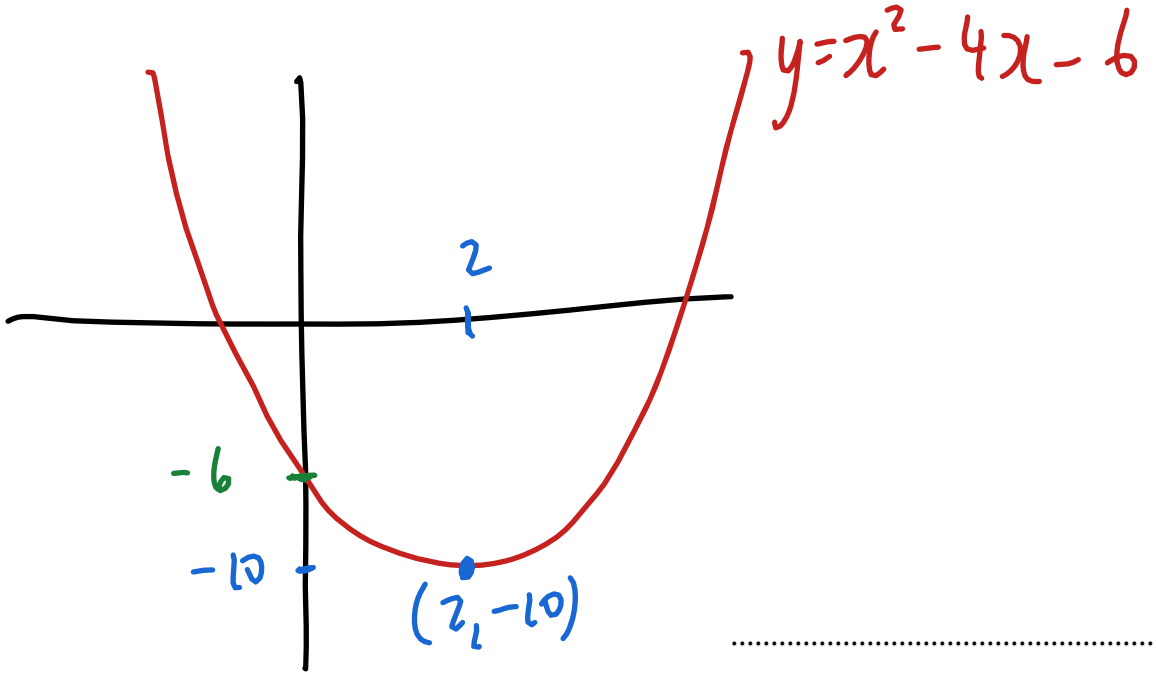
(2 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph $y = x^2 - 4x - 6$

$(2, -10)$

(1 mark)

(c) Hence sketch the curve $y = x^2 - 4x - 6$



(2 marks)

4. (a) Write $x^2 - 8x + 8$ in the form $(x + a)^2 + b$ where a and b are integers

$(x-4)^2 + 8 - 16$

$(x-4)^2 - 8$

(2 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph
$y = x^2 - 8x + 8$

$(4, -8)$

(1 mark)

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5. (a) Write $x^2 - 3x - 1$ in the form $(x + a)^2 + b$ where a and b are integers

$(x - \frac{3}{2})^2 - 1 - \frac{9}{4}$

$(x-3/2)^2 - 3.25$

(2 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph
$y = x^2 - 3x - 1$

$(3/2, -3.25)$

(1 mark)

6. (a) Write $2x^2 - 12x + 8$ in the form $a(x + b)^2 + c$ where a, b and c are integers

$2(x^2 - 6x + 4)$

$2[(x-3)^2 + 4 - 9]$

$2[(x-3)^2 - 5]$

$2(x-3)^2 - 10$

(3 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph
$y = 2x^2 - 12x + 8$

$(3, -10)$

(1 mark)

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7. (a) Write $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$ where a, b and c are integers

$2[x^2 - 6x + 11.5]$

$2[(x-3)^2 + 11.5 - 9]$

$2[(x-3)^2 + 2.5]$

$2(x-3)^2 + 5$

(3 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph
$y = 2x^2 - 12x + 23$

$(3, 5)$

(1 mark)

8. (a) Write $3x^2 - 6x + 6$ in the form $a(x + b)^2 + c$ where a, b and c are integers

$3[x^2 - 2x + 2]$

$3[(x-1)^2 + 2 - 1]$

$3[(x-1)^2 + 1]$

$3(x-1)^2 + 3$

(3 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph
$y = 3x^2 - 6x + 6$

$(1, 3)$

(1 mark)

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9. (a) Write $-x^2 + 6x + 10$ in the form $a(x + b)^2 + c$ where a, b and c are integers

$-1[x^2 - 6x - 10]$

$-1[(x-3)^2 - 10 - 9]$

$-1[(x-3)^2 - 19]$

$-(x-3)^2 + 19$

(3 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph
$y = 10 + 6x - x^2$

$(3, 19)$

(1 mark)

10. (a) Write $1 - 6x - x^2$ in the form $a(x + b)^2 + c$ where a, b and c are integers

$-1[-1 + 6x + x^2]$

$-1[x^2 + 6x - 1]$

$-1[(x+3)^2 - 1 - 9]$

$-1[(x+3)^2 - 10]$

$-(x+3)^2 + 10$

(3 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph
$y = 1 - 6x - x^2$

$(-3, 10)$

(1 mark)

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11. (a) Write $-3x^2 + 12x - 9$ in the form $a(x + b)^2 + c$ where a, b and c are integers

$-3[x^2 - 4x + 3]$

$-3[(x-2)^2 + 3 - 4]$

$-3[(x-2)^2 - 1]$

$-3(x-2)^2 + 3$

$-3(x-2)^2 + 3$

(3 marks)

(b) Hence or otherwise write down the coordinates of the turning point of the graph
$y = -3x^2 + 12x - 9$

$(2, 3)$

(1 mark)

12. Solve the equation $x^2 + 8x + 10 = 0$ by completing the square. Give your answers in surd form

$(x+4)^2 + 10 - 16 = 0$

$(x+4)^2 - 6 = 0$

$(x+4)^2 = 6$

$x+4 = \pm\sqrt{6}$

$x = -4 \pm \sqrt{6}$

.....

(5 marks)

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13. Solve the equation $x^2 + 4x - 2 = 0$ by completing the square. Give your answers in surd form

$(x+2)^2 - 2 - 4 = 0$

$(x+2)^2 - 6 = 0$

$(x+2)^2 = 6$

$x+2 = \pm\sqrt{6}$

$x = -2 \pm \sqrt{6}$

.....

(5 marks)

14. Solve the equation $2x^2 - 8x + 7 = 0$. Give your answers in surd form

$2\left[x^2 - 4x + \frac{7}{2}\right] = 0$

$2\left[(x-2)^2 + \frac{7}{2}\right] = 0$

$2\left[(x-2)^2 + \frac{7}{2} - 4\right] = 0$

$2\left[(x-2)^2 - 0.5\right] = 0$

$2(x-2)^2 - 1 = 0$

$2(x-2)^2 = 1$

$(x-2)^2 = \frac{1}{2}$

$x - 2 = \pm\sqrt{\frac{1}{2}}$

$x = 2 \pm \sqrt{\frac{1}{2}}$

.....

(5 marks)

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15. Solve the equation $x^2 + 7x + 2.25 = 0$. Give your answers in surd form

$x^2 + 7x + \frac{9}{4} = 0$

$\left(x + \frac{7}{2}\right)^2 + \frac{9}{4} - \frac{49}{4} = 0$

$\left(x + \frac{7}{2}\right)^2 - \frac{40}{4} = 0$

$\left(x + \frac{7}{2}\right)^2 = 10$

$\left(x + \frac{7}{2}\right) = \pm\sqrt{10}$

$x = -\frac{7}{2} \pm \sqrt{10}$

.....

(5 marks)

16. The expression $x^2 - 3x + 8$ can be written in the form $(x - a)^2 + b$ for all values of x.

(i) Find the value of a and the value of b.

$(x - 3/2)^2 + 8 - 9/4$

$(x - 3/2)^2 + 23/4$

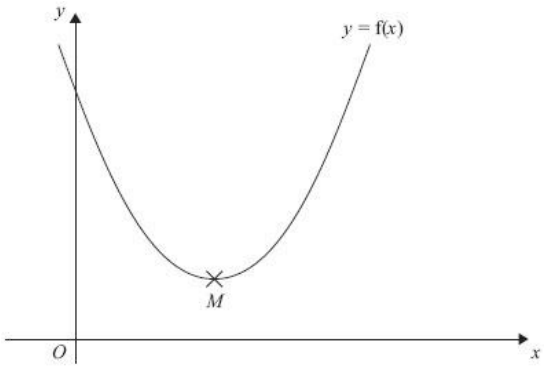
$a = 3/2 \quad b = 23/4$

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(3 marks)

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 3x + 8$

The diagram shows part of a sketch of the graph of $y = f(x)$.



The minimum point of the curve is M.

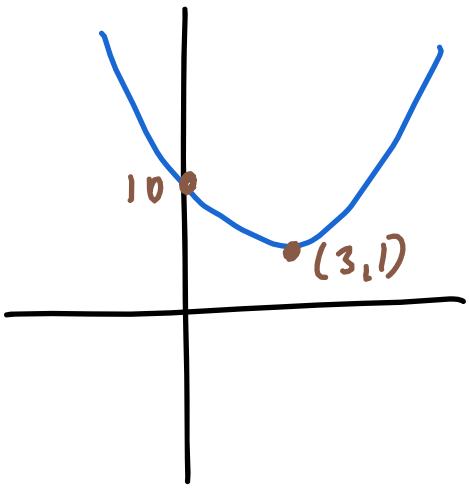
(ii) Write down the coordinates of M.

$(3/2, 23/4)$

.....

(1 mark)

17. (i) Sketch the graph of f(x) = x^2 – 6x + 10, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.



$x^2 - 6x + 10$

$(x-3)^2 + 10 - 9$

$(x-3)^2 + 1$

.....

(4 marks)

(ii) Hence, or otherwise, determine whether f(x) − 3= 0 has any real roots. Give reasons for your answer.

No roots.
Minimum point is above the x axis.

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(2 marks)

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18. Write $2 + 0.8x - 0.04x^2$ in the form $A - B(x + C)^2$, where A, B and C are constants to be found.

$-0.04(x^2 - 20x - 50)$

$-0.04[(x-10)^2 - 150]$

$-0.04(x-10)^2 + 6$

$6 - 0.04(x-10)^2$

.....

(4 marks)

19. The minimum point of a quadratic curve is (1, −4). The curve cuts the *y*-axis at −1.
Show that the equation of the curve is $y = 3x^2 - 6x - 1$

$y = 3(x - 1)^2 - 4$

$y = 3(x^2 - 2x + 1) - 4$

$y = 3x^2 - 6x + 3 - 4$

$y = 3x^2 - 6x - 1$

.....

(5 marks)

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20. The maximum point of a quadratic curve is (−2, −5). The curve cuts the *y*-axis at −13.
Find the equation of the curve. Give your answer in the form $ax^2 + bx + c$.

$y = -2(x + 2)^2 - 5$

$y = -2(x^2 + 2x + 4) - 5$

$y = -2x^2 - 4x - 8 - 5$

$y = -2x^2 - 4x - 13$

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(5 marks)

TOTAL FOR PAPER IS 92 MARKS