Solutions by
Kenneth Stafford
MATHS TUTOR
ken@kennethstafford.co.uk

GCSE (9-1) Grade 6 Expanding Triple Brackets

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
  - there may be more space than you need.
- **Show all your working out**

Information

- The total mark for this paper is 60.
- The marks for **each** question are shown in brackets.
  - use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed

Advice

- Read each question carefully before you start to answer it
- Attempt every question
- Check your answers if you have time at the end

1. Expand and Simplify $(x+1)(x+3)(x+4)$

$(x+3)(x+4) = x^2 + 7x + 12$

$(x+1)(x^2+7x+12) = x^3 + 7x^2 + 12x$

$x^2 + 7x + 12$

$x^3 + 8x^2 + 19x + 12$

....

(Total 3 marks)

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2. Expand and Simplify $(2x+5)(3x+7)(x+5)$

$(2x+5)(3x+7) = 6x^2 + 29x + 35$

$(x+5)(6x^2+29x+35) = 6x^3 + 29x^2 + 35x$

$30x^2 + 145x + 175$

$6x^3 + 59x^2 + 180x + 175$

....

(Total 3 marks)

3. Expand and Simplify $(x-3)(x-1)(2x-3)$

$(2x-3)(x-3) = 2x^2-9x+9$

$(x-1)(2x^2-9x+9) = 2x^3-9x^2+9x$

$\quad\quad -2x^2+9x-9$

$2x^3-11x^2+18x-9$

.....

(Total 4 marks)

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4. Expand and Simplify $(2x+3)(x-2)(3x-1)$

$(2x+3)(3x-1) = 6x^2+7x-3$

$(x-2)(6x^2+7x-3) = 6x^3+7x^2-3x$

$\quad\quad -12x^2-14x+6$

$6x^3-5x^2-17x+6$

.....

(Total 4 marks)

5. Expand and Simplify $(x+4)^2(3x-7)$

$(x+4)(x+4) = x^2+8x+16$

$(3x-7)(x^2+8x+16) = 3x^3+24x^2+48x$

$-7x^2-56x-112$

$3x^3+17x^2-8x-112$

.....

(Total 4 marks)

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6. Expand and Simplify $(5x+3)(2x-1)^2$

$(2x-1)(2x-1) = 4x^2-4x+1$

$(5x+3)(4x^2-4x+1) = 20x^3-20x^2+5x$

$12x^2-12x+3$

$20x^3-8x^2-7x+3$

.....

(Total 4 marks)

7. Show that $(2x + 3)^3 = 8x^3 + 36x^2 + 54x + 27$ for all values of x.

$(2x+3)(2x+3)(2x+3)$

$(2x+3)(2x+3) = 4x^2+12x+9$

$(2x+3)(4x^2+12x+9) =$ $8x^3 + 24x^2 + 18x$
$12x^2 + 36x + 27$

$8x^3 + 36x^2 + 54x + 27$

.....

(Total 4 marks)

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8. Show that $(x - 4)^2(x + 3)$ simplifies to $x^3 + ax^2 + bx + c$ where a, b and c are integers.

$(x-4)(x-4) = x^2 - 8x + 16$

$(x+3)(x^2-8x+16) = x^3 - 8x^2 + 16x$
$3x^2 - 24x + 48$

$x^3 - 5x^2 - 8x + 48$

.....

(Total 4 marks)

9. Express $(3x - 1)^3$ in the form $ax^3 + bx^2 + cx + d$ where a, b, c, d are integers.

$(3x-1)(3x-1)(3x-1)$

$(3x-1)(3x-1) = 9x^2 - 6x + 1$

$(3x-1)(9x^2-6x+1) = 27x^3 - 18x^2 + 3x$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad - 9x^2 + 6x - 1$

$27x^3 - 27x^2 + 9x - 1$

(Total 4 marks)

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10. $(3x + 5)(x - 4)(3x - 2) = 9x^3 + Ax^2 + Bx + 40$

Work out the value of A and the value of B.

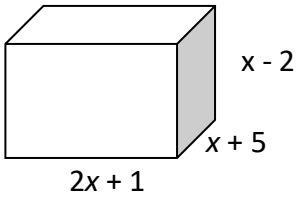
$(3x+5)(3x-2) = 9x^2 + 9x - 10$

$(x-4)(9x^2+9x-10) = 9x^3 + 9x^2 - 10x$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad -36x^2 - 36x + 40$

$9x^3 - 27x^2 - 46x + 40$

(Total 4 marks)

11. Here is a cuboid.



All measurements are in centimetres.

Show that the volume of the cuboid is $(2x^3 + 7x^2 - 17x - 10)$ cm^3

$(2x+1)(x-2)(x+5)$

$(2x+1)(x-2) = 2x^2 - 4x + x - 2$

$(x+5)(2x^2 - 3x - 2) = 2x^3 - 3x^2 - 2x$
$\quad 10x^2 - 15x - 10$
$2x^3 + 7x^2 - 17x - 10$ cm^3

(Total 4 marks)

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12. A cuboid has dimensions $x + 2$ cm, $2x - 1$ cm and $2x + 3$ cm.
Show that the volume of the cuboid is $4x^3 + 12x^2 + 5x - 6$ cm^3

$(x+2)(2x-1)(2x+3)$

$(2x-1)(2x+3) = 4x^2 + 6x - 2x - 3$

$(x+2)(4x^2 + 4x - 3) = 4x^3 + 4x^2 - 3x$
$\quad 8x^2 + 8x - 6$
$4x^3 + 12x^2 + 5x - 6$

(Total 4 marks)

13. (a) Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c

$(x+3)(x^2+bx+c) = x^3 - x^2 - 17x - 15$

$3c = -15$

$c = -5$

$(x+3)(x^2+bx-5) = x^3 - x^2 - 17x - 15$

$-5x + 3bx = -17x$

$3b = -12$

$b = -4$

$b = -4 \quad c = -5$

(Total 4 marks)

(b) Hence fully factorise $x^3 - x^2 - 17x - 15$

$(x+3)(x^2-4x-5)$

$(x+3)(x-5)(x+1)$

(Total 2 marks)

14. Given that $(2x + 5y)(3x - y)(2x + y) = ax^3 + bx^2y + cx^2y + dy^3$ where a, b, c, d are constants, find the values of a, b, c and d.

$(2x+5y)(3x-y)(2x+y)$

$(2x+5y)(3x-y) = 6x^2 - 2xy + 15xy - 5y^2$

$(2x+y)(6x^2+13xy-5y^2) = 12x^3 + 26x^2y - 10xy^2$
$6x^2y + 13xy^2 - 5y^3$

$12x^3 + 32x^2y + 3xy^2 - 5y^3$

(Total 4 marks)

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15. The smallest of three consecutive positive odd numbers is $(2x - 1)$.

Work out the product of the three numbers.
Give your answer in the form $ax^3 + bx^2 + cx + d$.

$(2x-1)(2x+1)(2x+3)$

$(2x+3)(2x-1) = 4x^2 + 4x - 3$

$(2x+1)(4x^2+4x-3) = 8x^3 + 8x^2 - 6x$
$4x^2 + 4x - 3$
$8x^3 + 12x^2 - 2x - 3$

(Total 4 marks)

TOTAL FOR PAPER IS 60 MARKS