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GCSE (9-1) Grade 7 Composite and Inverse Functions

Unfinished. Please support to inspire me on.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Show all your working out**

Information

- The total mark for this paper is 148.
- The marks for **each** question are shown in brackets.
  – use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed

Advice

- Read each question carefully before you start to answer it
- Attempt every question
- Check your answers if you have time at the end

1. Functions f and g is such that $f(x) = 2x - 1$ and $g(x) = \frac{3}{x}$

(a) Find the value of

(i) f(3)

$2(3) - 1 = 5$

..... (1)

(ii) fg(6)

$g(6) = \frac{3}{6} = \frac{1}{2}$

$f(\frac{1}{2}) = 2(\frac{1}{2}) - 1 = 0$

..... (2)

(b) $f^{-1}(x) =$

$y = 2x - 1$

$x = 2y - 1$

$2y = x + 1$

$y = \frac{x+1}{2} = f^{-1}(x)$

$f^{-1}(x) = \frac{x+1}{2}$

..... (2)

(c) $gf(x) =$

$\frac{3}{2x-1}$

..... (2)

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2. The function f is such that $f(x) = 4x - 1$

(a) Find $f^{-1}(x)$

..... (2)

The function g is such that $g(x) = kx^2$ where k is a constant.

(b) Given that $fg(2) = 12$, work out the value of k

..... (2)

3. Functions f and g are such that $f(x) = 3(x - 4)$ and $g(x) = \frac{x}{5} + 1$

(a) Find the value of f(10)

…….. (1)

(b) Find $g^{-1}(x)$

…….. (2)

(c) Show that $ff(x) = 9x - 48$

…….. (2)

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4. Given that $f(x) = x^2$ and $g(x) = x - 6$, solve the equation $fg(x) = g^{-1}(x)$

…….. (3)

5. f and g are functions such that $f(x) = 2x - 3$ and $g(x) = 1 + \sqrt{x}$

(a) Calculate $f(-4)$

…….. (1)

(b) Given that $f(a) = 5$, find the value of a.

…….. (2)

(c) Calculate $gf(6)$.

…….. (2)

(d) Find the inverse function $g^{-1}(x)$.

…….. (2)

6. Functions f and g are such that

$f(x) = \dfrac{1}{x+2}$ and $g(x) = \sqrt{x-1}$

(a) Calculate fg(10)

…….. (2)

(b) Find the inverse function $g^{-1}(x)$.

…….. (2)

7. Functions f and g are such that

f(x) = $2x + 2$ and g(x) = $2x^2 - 5$

(a) Find the composite function fg.
Give your answer as simply as possible.

…….. (2)

(b) Find the inverse function $f^{-1}(x)$.

…….. (2)

(c) Hence, or otherwise, solve $f^{-1}(x) = g^{-1}(x)$.

…….. (3)

8. The function f is such that $f(x) = \dfrac{1}{x+3}$

(a) Find the value of f(2)

…….. (1)

(b) Given that $f(a) = \dfrac{1}{10}$, find the value of a.

…….. (2)

The function g is such that $g(x) = x + 2$

(c) Find the function gf.

Give your answer as a single algebraic fraction in its simplest form.

…….. (2)

9. Functions f and g are such that $f(x) = x^2$ and $g(x) = x - 3$

(a) Find $gf(x)$.

…….. (2)

(b) Find the inverse function $g^{-1}(x)$.

…….. (2)

(c) Solve the equation $gf(x) = g^{-1}(x)$.

…….. (3)

10. The function f is such that $f(x) = (x - 1)^2$

(a) Find f(8)

…….. (1)

The function g is such that $g(x) = \dfrac{x}{x-1}$

(b) Solve the equation $g(x) = 1.2$

…….. (3)

(c) (i) Express the inverse function g^{-1} in the form $g^{-1}(x) =$

…….. (2)

(ii) Hence write down $gg(x)$ in terms of x.

…….. (2)

11. f is a function such that $f(x) = \dfrac{1}{x^2+1}$

(a) Find $f(\frac{1}{2})$

…….. (1)

g is a function such that $g(x) = \sqrt{x-1}$, $x \geq 1$

(b) Find $fg(x)$
Give your answer as simply as possible.

…….. (3)

12. The function f is such that $f(x) = \frac{x-6}{2}$

(a) Find f(8)

…….. (1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) =$

…….. (2)

The function g is such that $g(x) = \sqrt{x-4}$

(c) Express the function gf in the form $gf(x) =$
Give your answer as simply as possible.

…….. (3)

13. Functions f and g are such that $f(x) = 3x - 2$ and $g(x) = \dfrac{10}{x+2}$

(a) Express the inverse function f^{-1} in the form $f^{-1}(x) =$

…….. (2)

(b) Find $gf(x)$
Simplify your answer.

…….. (2)

14. Functions f and g are such that f(x) = $\frac{2}{x}$ and g(x) = $\frac{x+1}{x}$

(a) Solve gf(a) = 3

…….. (3)

(b) Express the inverse function g^{-1} in the form $g^{-1}(x)$ =

…….. (3)

15. Functions g and h are such that $g(x) = \frac{x}{2x-5}$ and $h(x) = x + 4$

(a) Find the value of g(1)

…….. (1)

(b) Find gh(x)
Simplify your answer.

…….. (2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) =$

…….. (3)

16. The function f is such that $f(x) = \frac{1}{x+4}, \quad x \neq -4.$

Evaluate $f^{-1}(3)$.

…….. (3)

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17. Given that $f(x) = 3x - 1$, $g(x) = x^2 + 4$ and $fg(x) = gf(x)$,

show that $x^2 - x - 1 = 0$

…….. (5)

18. Functions f and g are such that

$f(x) = 3x + 2$ $\quad\quad$ $g(x) = x^2 + 1$

Find an expression for $(fg)^{-1}(x)$

…….. (3)

19. The function f is such that $f(x) = \dfrac{8}{x+2}$

(a) Find $f^{-1}(x)$.

…….. (2)

(b) Solve the equation $f^{-1}(x) = f(x)$

…….. (3)

20. The function g is such that $g(x) = \frac{1}{1-x}$ for $x \neq 1$

(a) Prove that $gg(x) = \frac{x-1}{x}$

…….. (3)

(b) Find ggg(3)

…….. (3)

21. Functions f, g and h are such that $f(x) = 3 - x$, $g(x) = x^2 - 14$ and $h(x) = x - 2$

Given that $f(x) = gfh(x)$, find the values of x.

…….. (5)

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22. The function f is defined by $f(x) = \frac{x-1}{x}$, $x \neq 0$

Solve $ff(x) = -2$

…….. (4)

23. $f(x) = \dfrac{x}{x+3}, \quad x \in \mathbb{R}, \ x \neq -3$

(a) If $f^{-1}(x) = -5$, find the value of x.

…….. (3)

(b) Show that $ff^{-1}(x) = x$

…….. (3)

24. $f(x) = \dfrac{x}{x+3}, \quad x \in \mathbb{R}, \ x \neq -3$

(a) If $f^{-1}(x) = -5$, find the value of x.

…….. (3)

(b) Show that $ff^{-1}(x) = x$

…….. (3)

25. $f(x) = 2x + c$

$g(x) = cx + 5$

$fg(x) = 6x + d$

c and d are constants.

Work out the value of d.

…….. (3)

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26. f(x) = $3x^2 - 2x - 8$

Express f(x + 2) in the form $ax^2 + bx$

…….. (3)

27. $f(x) = x^2 - 2x - 4$

Express $f(2x - 1)$ in the form $ax^2 + bx + c$

…….. (3)

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28. $f(x) = x^2 + 3x + 4$

Show that $f(x - 2) - f(x) = -4x - 2$

…….. (3)

TOTAL FOR PAPER: 148 MARKS