Name:.....

Total Marks:.....

GCSE (9-1) Grade 8/9 Nth Term of a Quadratic Sequence

Naiker Solutions

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
  - there may be more space than you need.
- **Show all your working out**

Information

- The total mark for this paper is 85.
- The marks for **each** question are shown in brackets.
  - use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed

Advice

- Read each question carefully before you start to answer it
- Attempt every question
- Check your answers if you have time at the end

1. Write down the next two terms in the following sequence

6, 9, 14, 21, 30, 41

+3 +5 +7 +9 +11

30 and 41

(2 marks)

---

2. Write down the next two terms in the following sequence

4, -5, -18, -35, -56, -81

-9 -13 -17 -21 -25

-56 and -81

(2 marks)

3. The nth term of a sequence is

$2n^2 + 3n - 2$

Work out the 8th term of the sequence

$n = 8 \Rightarrow 2(8)^2 + 3(8) - 2$

$= 2(64) + 24 - 2$

$= 128 + 24 - 2$

$= 150$

150

(2 marks)

4. The nth term of a sequence is

$n^2 + 3n - 1$

Write down the first 5 terms in the sequence

1st term $\Rightarrow n = 1 \Rightarrow (1)^2 + 3(1) - 1 = 3$

2nd term $\Rightarrow n = 2 \Rightarrow (2)^2 + 3(2) - 1 = 9$

3rd term $\Rightarrow n = 3 \Rightarrow (3)^2 + 3(3) - 1 = 17$

4th term $\Rightarrow n = 4 \Rightarrow (4)^2 + 3(4) - 1 = 27$

5th term $\Rightarrow n = 5 \Rightarrow (5)^2 + 3(5) - 1 = 39$

3, 9, 17, 27, 39

(2 marks)

5. Find an expression, in terms of n, for the nth term of the quadratic sequence:

2, 3, 6, 11, 18

+1 +3 +5 +7

+2 +2 +2 $\Rightarrow \frac{2}{2} = 1$ $\therefore n^2$

n^2 1 4 9 16 25

↓+1 ↓-1 ↓-3 ↓-5 ↓-7

2 3 6 11 18

+1, -1, -3, -5, -7

Nth term = $-2n + 3$ of linear sequence

$\therefore$ Nth term of Quadratic sequence = $n^2 - 2n + 3$

$n^2 - 2n + 3$

(4 marks)

6. Find an expression, in terms of n, for the nth term of the quadratic sequence:

3, 10, 19, 30, 43

+7 +9 +11 +13

+2 +2 +2 $\therefore n^2$

n^2 1 4 9 16 25

↓+2 ↓+6 ↓+10 ↓+14 ↓+18

3 10 19 30 43

2, 6, 10, 14, 18 $\Rightarrow$ nth term = $4n - 2$

+4 +4 +4 +4

$\therefore$ nth term of sequence = $n^2 + 4n - 2$

$n^2 + 4n - 2$

(4 marks)

7. Find an expression, in terms of n, for the nth term of the quadratic sequence:

-1, 0, 3, 8, 15

+1 +3 +5 +7

+2 +2 +2

$\therefore n^2$

n^2: 1 4 9 16 25

-2 -4 -6 -8 -10

-1 0 3 8 15

-2, -4, -6, -8, -10 $\Rightarrow -2n + 0$

$\therefore$ Nth term is $n^2 - 2n$

$n^2 - 2n$

(4 marks)

---

8. Find an expression, in terms of n, for the nth term of the quadratic sequence:

-2, 4, 14, 28, 46

+6 +10 +14 +18

+4 +4 +4

$\therefore 2n^2$ $(4 \div 2)$

$2n^2$: 2 8 18 32 50

-4 -4 -4 -4 -4

-2 4 14 28 46

$\therefore$ Nth term is $2n^2 - 4$

$2n^2 - 4$

(4 marks)

9. Find an expression, in terms of n, for the nth term of the quadratic sequence:

6, 10, 12, 12, 10

+4 +2 +0 −2

−2 −2 −2 ⇒ $-n^2$ $(-2 \div 2)$

$-n^2$ −1 −4 −9 −16 −25

+7 +14 +21 +28 +35

6 10 12 12 10

7, 14, 21, 28, 35 ⇒ $7n - 0$

+7 +7 +7 +7

∴ N^{th} term is $-n^2 + 7n$ $-n^2 + 7n$

(4 marks)

---

10. Find an expression, in terms of n, for the nth term of the quadratic sequence:

-2, -1, 1, 4, ...

+1 +2 +3

+1 +1 ⇒ $0.5n^2$ $(1 \div 2)$

$0.5n^2$ 0.5 2 4.5 8

−2.5 −3 −3.5 −4

−2 −1 1 4

−2.5, −3, −3.5, −4 ⇒ $-0.5n - 2$

−0.5 −0.5 −0.5

∴ N^{th} term is $0.5n^2 - 0.5n - 2$ $0.5n^2 - 0.5n - 2$

(4 marks)

11. Here are the first 6 terms of a sequence.

9 4 -5 −18 −35

(a) Find an expression, in terms of *n*, for the *n*th term of this sequence.

9 4 -5 -18 -35

-5 -9 -13 -17

-4 -4 -4 $\Rightarrow -2n^2$ $(-4 \div 2)$

$-2n^2$ -2 -8 -18 -32 -50

+11 +12 +13 +14 +15

9 4 -5 -18 -35

11, 12, 13, 14, 15 $\Rightarrow n+10$

+1 +1 +1 +1

$\therefore$ N^{th} term is $-2n^2+n+10$

$-2n^2+n+10$

(4 marks)

(b) Hence find the term that has value −1215

$-2n^2+n+10 = -1215$

$-2n^2+n+10+1215 = 0$

$-2n^2+n+1225 = 0$

$(\times -1)$ $2n^2 - n - 1225 = 0$

a b c

Using $n = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$n = \dfrac{1 \pm \sqrt{(-1)^2-4(2)(-1225)}}{2(2)}$

$n = 25$ or -24.5

Invalid

$\therefore$ 25th term

25th

(2 marks)

12. Here are the first 5 terms of a sequence.

7 16 29 46 67

(a) Find an expression, in terms of *n*, for the *n*th term of this sequence.

7 16 29 46 67

9 13 17 21

4 4 4 $\Rightarrow 2n^2$

$2n^2$

2 8 18 32 50

+5 +8 +11 +14 +17

7 16 29 46 67

5, 8, 11, 14, 17 $\Rightarrow 3n - 2$

+3 +3 +3 +3

$\therefore$ N^{th} term is $2n^2 + 3n + 2$

$2n^2 + 3n + 2$

(4 marks)

(b) Hence find the term that has value 862

$2n^2 + 3n + 2 = 862$

$2n^2 + 3n - 86 = 0$

$a = 2, \quad b = 3, \quad c = -86$

Using $n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$n = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-86)}}{2(2)}$

$n = 20$ or -21.5

NOT VALID

$\therefore$ 20^{th} term

20^{th}

(2 marks)

13. Here are the first 5 terms of a sequence.

3, 9, 17, 27, 39

+6, +8, +10, +12

+2, +2, +2 $\Rightarrow n^2$

Claire says that 161 is a term of this sequence

(a) Is Claire correct? Give a reason for your answer

n^2: 1, 4, 9, 16, 25

+2, +5, +8, +11, +14

3, 9, 17, 27, 39

2, 5, 8, 11, 14

+3, +3, +3, +3 $\Rightarrow 3n - 1$

$\therefore$ Nth term $= n^2 + 3n - 1$

$n^2 + 3n - 1 = 161$

$n^2 + 3n - 162 = 0$

$a = 1$, $b = 3$, $c = -162$

$$n = \frac{-3 \pm \sqrt{3^2 - 4(1)(-162)}}{2(1)}$$

$n = 11.31..$ or $n = -14.31...$

Since n is NOT A WHOLE NUMBER, therefore 161 is not a term in this sequence

.....

(4 marks)

Rob says that all the terms in the sequence are odd numbers

(b) Is Rob correct? Give a reason for your answer.

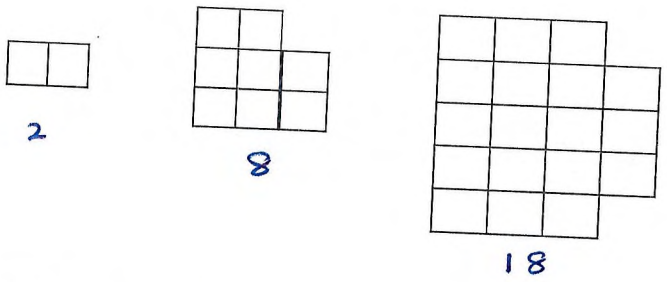
Rob is correct as all numbers are odd

3, 9, 17, 27, 39, 53, 69, 87, 107, 129.....

.....

(2 marks)

14. Here are some patterns made from square slabs.



(a) Write down an expression, in terms of n, for the nth term of this sequence.

2, 8, 18

+6 +10

+4 $\Rightarrow 2n^2$

$2n^2$

2 8 18

+0 +0 +0

2 8 18

$\therefore$ N^{th} term is $2n^2$

$2n^2$

.....

(4 marks)

(b) Jane says that 75 is a term in the quadratic sequence.

Is Jane correct? Give a reason for your answer.

$2n^2 = 75$

$n = \sqrt{\frac{75}{2}}$

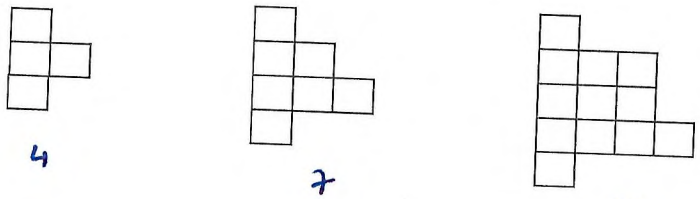
$n = 6.12$

NO, as n is not an integer

.....

(2 marks)

15. Here are some patterns made from tiles.



4 7 12

Find an expression, in terms of n, for the nth term of this sequence.

4, 7, 12

+3 +5

+2 $\Rightarrow n^2$

n^2 1 4 9

+3 +3 +3

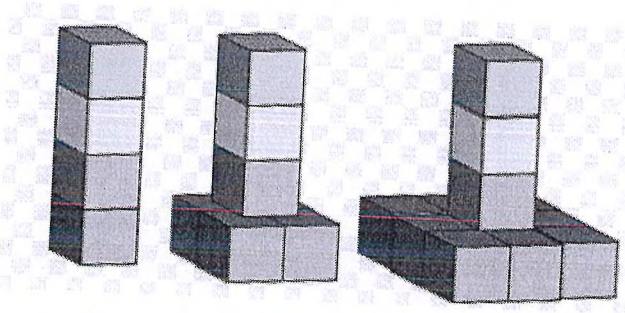
4 7 12

$\therefore$ N^{th} term is n^2+3

n^2+3

(4 marks)

16. Here are some patterns made from cubes.



4 7 12

Find an expression, in terms of n, for the nth term of this sequence.

4, 7, 12

+3 +5

+2 $\Rightarrow n^2$

n^2 1 4 9

↓+3 ↓+3 ↓+3

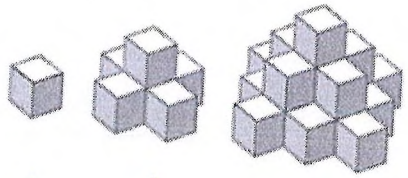
4 7 12

∴ N^{th} term is n^2+3

n^2+3

17. Here are some patterns made from cubes.

(4 marks)



1 6 19

(a) Work out the number of cubes needed to make the next pattern.

1, 6, 19, 40

+5 +13 +21

+8 +8

40

(2 marks)

(b) Find an expression, in terms of n, for the nth term of this sequence.

1, 6, 19, 40

+5 +13 +21

+8 +8 $\Rightarrow 4n^2$

$4n^2$ 4, 16, 36, 64

-3 -10 -17 -24

1, 6, 19, 40

3, 10, 17, 24

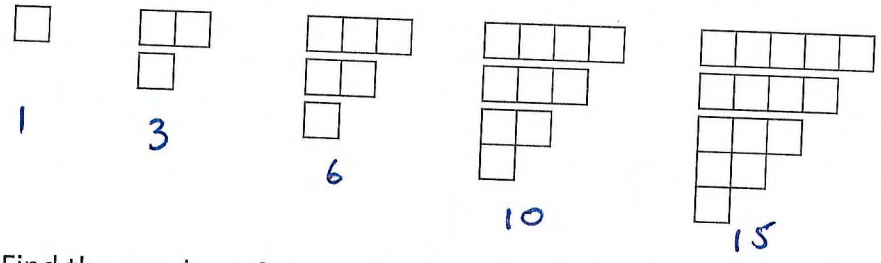
+7 +7 +7 $\Rightarrow 7n-4$

$\therefore$ Nth term $= 4n^2 + 7n - 4$

$4n^2 + 7n - 4$

(4 marks)

18. Here is a sequence of patterns made from centimetre squares.



Find the number of centimetre squares in the 100th pattern.

1 3 6 10 15

+2 +3 +4 +5

+1 +1 +1 $\Rightarrow \frac{1}{2}n^2$

$\frac{1}{2}n^2$ 0.5 2 4.5 8 12.5

+0.5 +1 +1.5 +2 +2.5

1 3 6 10 15

0.5, 1, 1.5, 2, 2.5

+0.5 +0.5 +0.5 +0.5 $\Rightarrow 0.5n - 0$

$\therefore$ Nth term $= \frac{1}{2}n^2 + \frac{1}{2}n$

or

$= 0.5n^2 + 0.5n$

100th pattern $\Rightarrow n = 100$

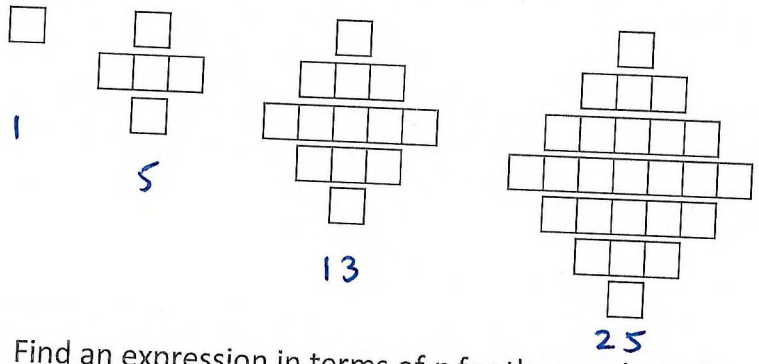
$\therefore 0.5(100)^2 + 0.5(100) = 5050$

$\frac{1}{2}n^2 + \frac{1}{2}n$

5050

(5 marks)

19. Here is a sequence of patterns made from centimetre squares.



Find an expression in terms of n for the number of centimetre squares in the nth pattern.

1 5 13 25

+4 +8 +12

+4 +4 $\Rightarrow 2n^2$

$2n^2$ 2 8 18 32

-1 -3 -5 -7

1 5 13 25

-1, -3, -5, -7

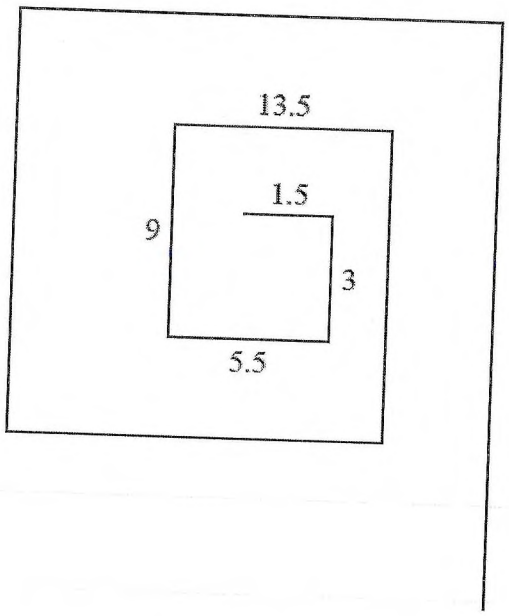
-2 -2 -2 $\Rightarrow -2n+1$

$\therefore$ n^{th} term $= 2n^2 - 2n + 1$

$2n^2 - 2n + 1$

(5 marks)

20. The diagram shows the first 10 sides of a spiral pattern.
It also gives the lengths, in cm, of the first 5 sides.



The lengths, in cm, of the sides of the spiral form a sequence.

Find an expression in terms of n for the length, in cm, of the nth side.

$1.5, 3, 5.5, 9, 13.5$

$+1.5 \quad +2.5 \quad +3.5 \quad +4.5$

$+1 \quad +1 \quad +1 \Rightarrow \frac{1}{2}n^2$

$\frac{1}{2}n^2 \quad \frac{1}{2}, 2, \frac{9}{2}, 8, \frac{25}{2}$

$+1 \downarrow \quad +1 \downarrow \quad +1 \downarrow \quad +1 \downarrow \quad +1 \downarrow \Rightarrow +1$

$1.5, 3, 5.5, 9, 13.5$

$\therefore N^{th}\ term = \frac{1}{2}n^2 + 1$

$\frac{1}{2}n^2 + 1$

.....

(3 marks)

TOTAL FOR PAPER IS 85 MARKS