Solutions by
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GCSE (9-1) Grade 8/9 Vectors Proof Questions

Unfinished. Please support to inspire me on.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Show all your working out**

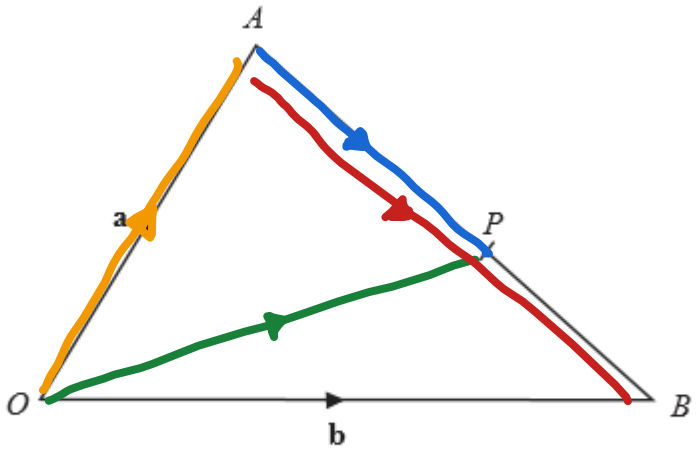
Information

- The total mark for this paper is 64.
- The marks for **each** question are shown in brackets.
  – use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed

Advice

- Read each question carefully before you start to answer it
- Attempt every question
- Check your answers if you have time at the end

1.



OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$

P is a point on AB so that $AP : PB$ is 2 : 3

Show that $\overrightarrow{OP} = \frac{1}{5}(3\mathbf{a} + 2\mathbf{b})$

$\overrightarrow{AB} = -a + b$

$\overrightarrow{AP} = \frac{2}{5}(-a + b) = -\frac{2}{5}a + \frac{2}{5}b$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = a + -\frac{2}{5}a + \frac{2}{5}b$

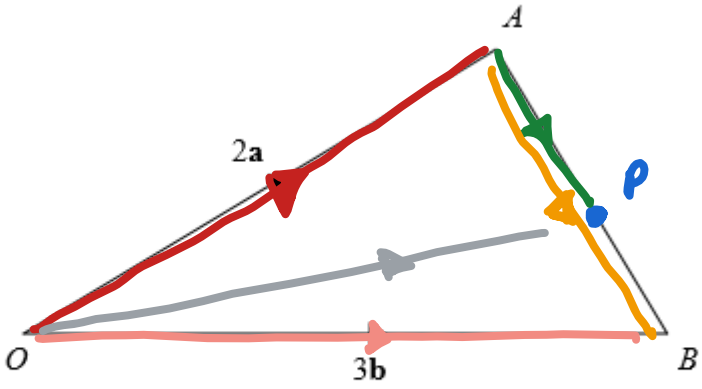
$= \frac{3}{5}a + \frac{2}{5}b$

$= \frac{1}{5}(3a + 2b)$

.....

(Total 3 marks)

2.



$\overrightarrow{OA} = 2\mathbf{a}$

$\overrightarrow{OB} = 3\mathbf{b}$

P is a point on *AB* so that *AP : PB* is 3 : 2

Show that $\overrightarrow{OP} = \frac{1}{5}(4\mathbf{a} + 9\mathbf{b})$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -2\mathbf{a} + 3\mathbf{b}$

$\overrightarrow{AB} = -2\mathbf{a} + 3\mathbf{b}$

$\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB}$

$= \frac{3}{5}(-2\mathbf{a} + 3\mathbf{b})$

$= -\frac{6}{5}\mathbf{a} + \frac{9}{5}\mathbf{b}$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

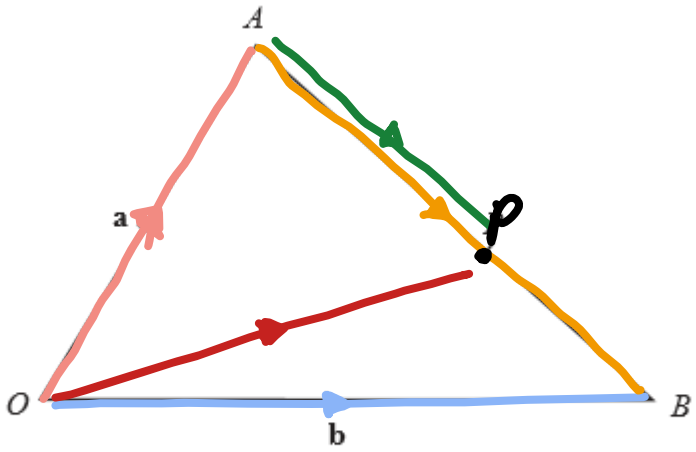
$= 2\mathbf{a} + -\frac{6}{5}\mathbf{a} + \frac{9}{5}\mathbf{b}$

$\overrightarrow{OP} = \frac{4}{5}\mathbf{a} + \frac{9}{5}\mathbf{b} = \frac{1}{5}(4\mathbf{a} + 9\mathbf{b})$

.....

(Total 4 marks)

3.



OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$

P Is the point on AB such that $AP : PB$ is 3 : 5

Find $\overrightarrow{OP}$ in terms of a and b. Give your answer in its simplest form

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -a + b$

$\overrightarrow{AB} = -a + b$

$\overrightarrow{AP} = \frac{3}{8}(\overrightarrow{AB})$

$= \frac{3}{8}(-a + b)$

$\overrightarrow{AP} = -\frac{3}{8}a + \frac{3}{8}b$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

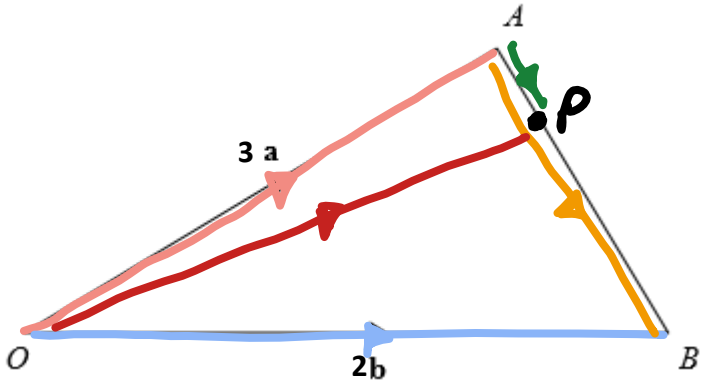
$= a + -\frac{3}{8}a + \frac{3}{8}b$

$\overrightarrow{OP} = \frac{5}{8}a + \frac{3}{8}b = \frac{1}{8}(5a + 3b)$

.....

(Total 4 marks)

4.



OAB is a triangle

$\overrightarrow{OA}$ = **3a**

$\overrightarrow{OB}$ = **2b**

P is a point on *AB* so that *AP : PB* is 1 : 3

Given that $\overrightarrow{OP} = k$ (9**a** + 2**b**)

Find the value of k

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -3a + 2b$

$\overrightarrow{AB} = -3a + 2b$

$\overrightarrow{AP} = \frac{1}{4}\overrightarrow{AB}$

$= \frac{1}{4}(-3a + 2b)$

$\overrightarrow{AP} = -\frac{3}{4}a + \frac{2}{4}b$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

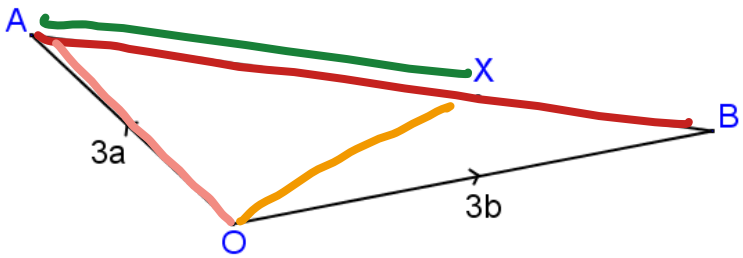
$= 3a + -\frac{3}{4}a + \frac{2}{4}b$

$\overrightarrow{OP} = \frac{9}{4}a + \frac{2}{4}b = \frac{1}{4}(9a + 2b)$ $k = \frac{1}{4}$

.....

(Total 4 marks)

5.



$\overrightarrow{OA}$ = **3a**
$\overrightarrow{OB}$ = **3b**

X is the point on *AB* such that *AX : XB* = 9 : 4

Find the value of k if $\overrightarrow{OX} = k\,(4a + 9b)$

$\overrightarrow{AB} = 3b - 3a$

$\overrightarrow{AX} = \frac{9}{13}(3b - 3a)$

$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$

$= 3a + \frac{9}{13}(3b - 3a)$

$= 3a + \frac{27}{13}b - \frac{27}{13}a$

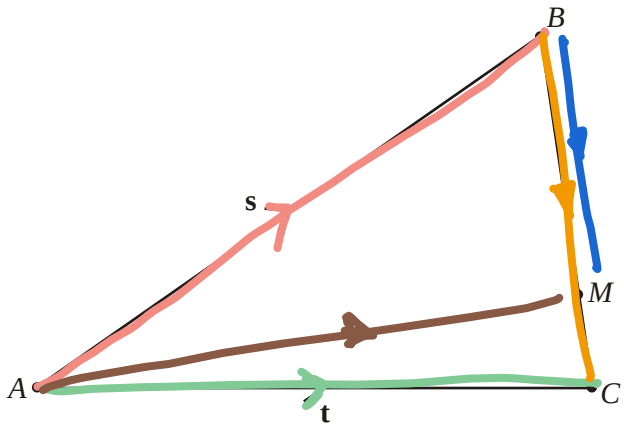
$= \frac{12}{13}a + \frac{27}{13}b$

$= \frac{3}{13}(4a + 9b)$

$k = \frac{3}{13}$

(Total 4 marks)

6.



Not drawn accurately

In triangle ABC, M lies on BC such that $BM = \frac{3}{4} BC$.

$\overrightarrow{AB} = \mathbf{s}$ and $\overrightarrow{AC} = \mathbf{t}$

Find $\overrightarrow{AM}$ in terms of **s** and **t**.

Give your answer in its simplest form.

$\overrightarrow{BC} = -s + t$

$\overrightarrow{BM} = \frac{3}{4}(-s+t)$

$= -\frac{3}{4}s + \frac{3}{4}t$

$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM}$

$= s + -\frac{3}{4}s + \frac{3}{4}t$

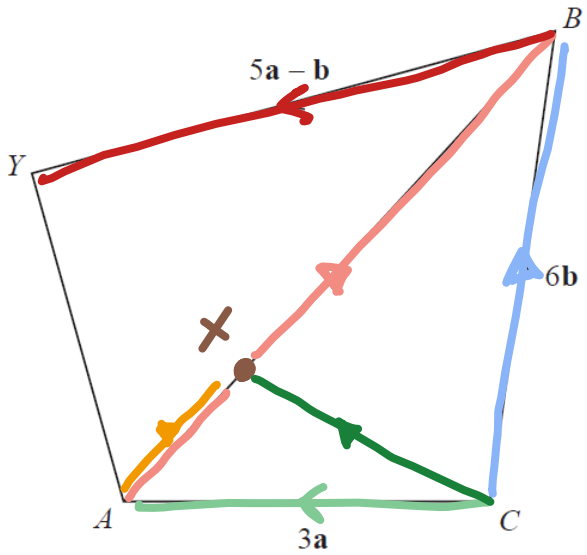
$= \frac{1}{4}s + \frac{3}{4}t$

$= \frac{1}{4}(s + 3t)$

$\overrightarrow{AM} = \frac{1}{4}(s + 3t)$

(Total 4 marks)

7.



CAYB is a quadrilateral.

$\overrightarrow{CA} = 3\mathbf{a}$

$\overrightarrow{CB} = 6\mathbf{b}$

$\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

X is the point on *AB* such that *AX* : *XB* = 1 : 2

Prove that $\overrightarrow{CX} = \frac{2}{5}\overrightarrow{CY}$

$\overrightarrow{AB} = -3a + 6b$

$\overrightarrow{AX} = \frac{1}{3}(-3a + 6b)$

$= -a + 2b$

$\overrightarrow{CX} = 3a + -a + 2b$

$= 2a + 2b$

$\overrightarrow{CY} = \overrightarrow{CB} + \overrightarrow{BY}$

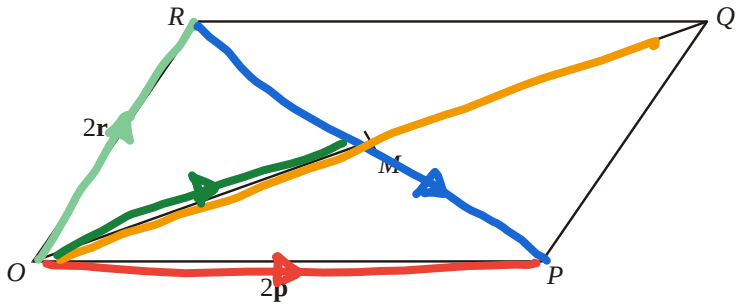
$6b + 5a - b$

$\overrightarrow{CY} = 5a + 5b$

$\frac{2}{5}\overrightarrow{CY} = \frac{2}{5}(5a+5b) = 2a+2b = \overrightarrow{CX}$

(Total 4 marks)

8.



OPQR is a parallelogram.

M is the mid-point of the diagonal *OQ*.

$\overrightarrow{OP} = 2\mathbf{p}$ and $\overrightarrow{OR} = 2\mathbf{r}$

$\overrightarrow{RP} = 2p - 2r$

Use vectors to prove that *M* is also the mid-point of *PR*.

$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$\overrightarrow{OQ} = 2p + 2r$

$\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OQ})$

$= \frac{1}{2}(2p + 2r)$

$\overrightarrow{OM} = p + r$

$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP}$

$2p = p + r + \overrightarrow{MP}$

$\overrightarrow{MP} = 2p - (p + r)$

$\overrightarrow{MP} = p - r$

$\overrightarrow{MP} = \frac{1}{2}\overrightarrow{RP}$

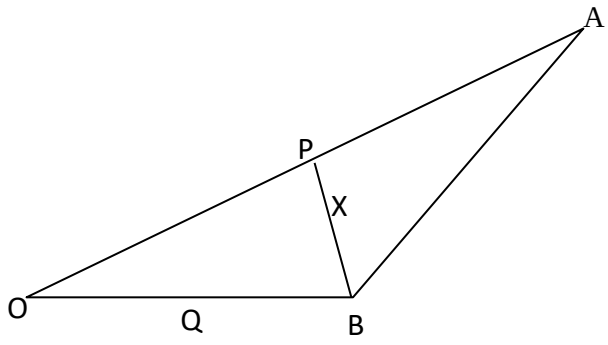
$\therefore$ M is the midpoint of PR

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(Total 4 marks)

9. OAB is a triangle. P and Q are the midpoints of OA and OB respectively.

The point X lies on the line PB, and PX:XB is in the ratio 1:2.



Show that $\overrightarrow{QX}$ is parallel to $\overrightarrow{QA}$

$\overrightarrow{OA} = 6a$

$\overrightarrow{OB} = 6b$

$\overrightarrow{OP} = 3a$

$\overrightarrow{QB} = 3b$

$\overrightarrow{PB} = -3a+6b$

$\overrightarrow{PX} = -a+2b$

$\overrightarrow{QX} = -3b+3a-a+2b$

$= 2a-b$

$\overrightarrow{BA} = -6b+6a$

$\overrightarrow{QA} = 3b-6b+6a$

$= 6a-3b$

$= 3(2a-b)$

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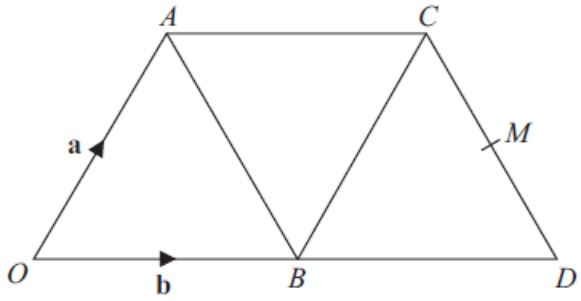
(Total 4 marks)

10. *OACD* is a trapezium made from three equilateral triangles.

$\overrightarrow{OA}$ = **a**

$\overrightarrow{OB}$ = **b**

M is the midpoint of *CD*.



(a) Write $\overrightarrow{AB}$ in terms of **a** and **b**.

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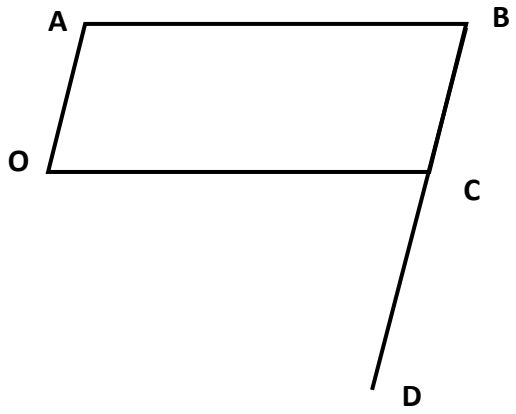
(1 mark)

(b) Show that $\overrightarrow{OC}$ is parallel to $\overrightarrow{BM}$.

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(3 marks)

11.



D is the point on *BC* extended such that *BC : CD* = 1 : 2

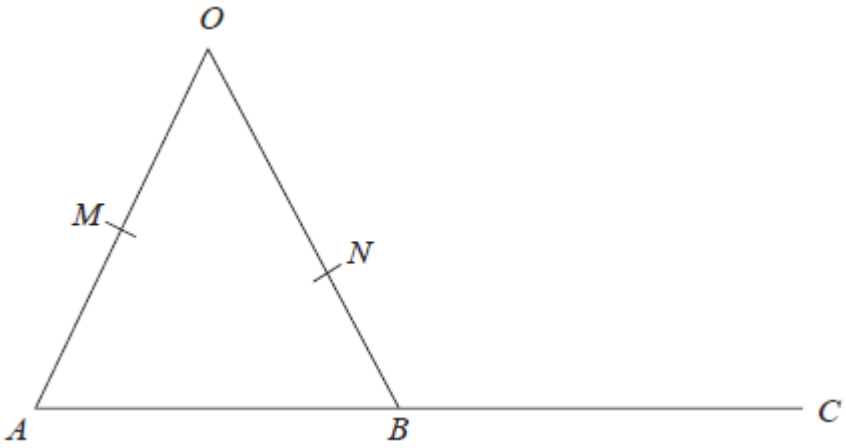
X is a point on *OC* such that $OX = \frac{1}{3} OC$

Show that A, X and D lie on the same straight line

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(Total 5 marks)

12.



OMA, ONB and *ABC* are straight lines.
M is the midpoint of *OA.*
B is the midpoint of *AC.*

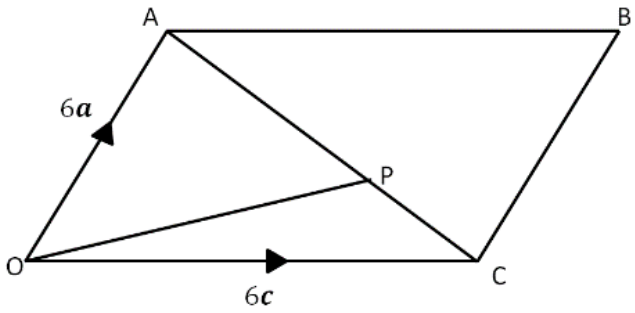
$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{ON} = k\mathbf{b}$ where *k* is a scalar quantity.

Given that *MNC* is a straight line, find the value of *k.*

.....

(Total 5 marks)

13.



$OABC$ is a parallelogram. P is the point on AC such that $AP = \frac{2}{3}AC$.

a) Find the vector $\overrightarrow{OP}$. Give your answer in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.

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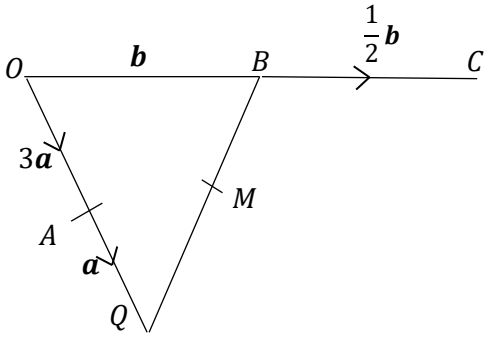
(Total 2 marks)

b) Given that the midpoint of CB is M, prove that OPM is a straight line.

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(Total 3 marks)

14.



$\overrightarrow{OA} = 3\boldsymbol{a}$ $\overrightarrow{AQ} = \boldsymbol{a}$ $\overrightarrow{OB} = \boldsymbol{b}$ $\overrightarrow{BC} = \frac{1}{2}\boldsymbol{b}$.

M is the midpoint of QB.

Prove that AMC is a straight line.

.....

(Total 5 marks)

TOTAL FOR PAPER: 64 MARKS